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SUMMARY

Formulas are presented expressing the major long period perturbations by the earth in the eccentricity and argument of perigee of small eccentricity satellites; the formulas include terms which involve $(J_i / J_2)^3$, where J_i is any odd zonal harmonic coefficient in the earth's gravitational potential. The perturbation expressions are then used to analyze the satellites Alouette 1 and Tiros 8.

ON THE LONG PERIOD PERTURBATIONS IN THE MOTION OF SMALL ECCENTRICITY SATELLITES

INTRODUCTION

Artificial earth satellites with small eccentricities ($\sim 10^{-3}$) pose interesting problems for those involved in orbit determination. The maindifficulty lies in the presence of eccentricity to some positive integral power in the denominators of trigonometric series representing perturbations in the orbital elements of satellites. These "small divisors" magnify the effects of the perturbations, and make necessary the inclusion of "higher-order" terms in order to obtain accurate orbit determination.

The discussion here will be limited to long-periodic effects on the eccentricity and argument of perigee caused by the odd zonal harmonics in the earth's gravitational potential. It is difficult to speak of the maximum "order" of the effects to be considered because of the presence of the small divisors – suffice it to say that terms involving up to $(J_i/J_2)^3$, where J_i is any odd zonal harmonic, will be included in the analysis.

Formulas for the perturbations in eccentricity and argument of perigee will be derived by two different methods – first, by a solution to Delaunay's Equations of Type II which appear in the expansion of his lunar theory, and then by the von Zeipel method. The two solutions, insofar as odd zonal harmonics are concerned, will be shown to be equivalent; the perturbation formulas will then be applied to the satellites Alouette 1 and Tiros 8.

A list of symbols appears in Appendix A.

DELAUNAY'S EQUATIONS OF TYPE II

Neglecting all terms multiplied by eccentricity to at least the first power, Delaunay's Equations of Type II are (see Reference 1)

$$\frac{de}{dt} = M \sin \theta$$

$$\frac{d\theta}{dt} = N + \frac{M}{e}\cos \theta, \tag{1}$$

where e is the eccentricity. With $\theta = g + \pi/2$, these equations applied to a close artificial satellite of the earth have

$$N = -\frac{3 J_2}{4 a_0^3 \sqrt{a_0}} (1 - 5 \cos^2 i_0) + \frac{3 J_2^2}{64 a_0^5 \sqrt{a_0}} (7 - 114 \cos^2 i_0 + 395 \cos^4 i_0)$$

$$- \frac{15 J_4}{32 a_0^5 \sqrt{a_0}} (3 - 36 \cos^2 i_0 + 49 \cos^4 i_0) + \cdots \tag{2}$$

$$M = \frac{3 J_3 \sin i_0}{8 a_0^4 \sqrt{a_0}} (1 - 5 \cos^2 i_0) + \frac{15 J_5 \sin i_0}{32 a_0^6 \sqrt{a_0}} (1 - 14 \cos^2 i_0 + 21 \cos^4 i_0)$$

$$+ \frac{105 J_7 \sin i_0}{1024 a_0^8 \sqrt{a_0}} (5 - 135 \cos^2 i_0 + 495 \cos^4 i_0 - 429 \cos^6 i_0)$$

$$+ \frac{315 J_9 \sin i_0}{4096 a_0^{10} \sqrt{a_0}} (7 - 308 \cos^2 i_0 + 2002 \cos^4 i_0 - 4004 \cos^6 i_0 + 2431 \cos^8 i_0)$$

$$+ \frac{3465 J_{11} \sin i_0}{131072 a_0^{12} \sqrt{a_0}} (21 - 1365 \cos^2 i_0 + 13650 \cos^4 i_0 - 46410 \cos^6 i_0$$

$$+ 62985 \cos^8 i_0 - 29393 \cos^{10} i_0) + \cdots , \tag{3}$$

where a_0 and i_0 are mean values of the semi-major axis and inclination, respectively. For illustrative purposes, only the even zonal harmonics through J_4 and the odd harmonics through J_{11} will be considered here; however, Equations (2) and (3) may include <u>all</u> the zonal harmonics.

With $Q = M/N$, solutions to Equations (1) of the form

$$e = e_1 + Q\beta_1 \cos\overline{\theta} + Q^2\beta_2 \cos 2\overline{\theta} + Q^3\beta_3 \cos 3\overline{\theta} + \cdots$$

$$\theta = \overline{\theta} + Q\,\alpha_1 \sin\overline{\theta} + Q^2\,\alpha_2 \sin 2\overline{\theta} + Q^3\,\alpha_3 \sin 3\overline{\theta} + \cdots, \tag{4}$$

where $\overline{\theta}$ is a linear function of time, are given in Reference 1. Modification of the solutions such that e_1 appears as the eccentricity constant results in

$$\begin{aligned}
\alpha_1 &= \frac{1}{e_1} + \frac{Q^2}{4e_1^3} \\
\beta_1 &= -1 + \frac{Q^2}{8e_1^2} \\
\alpha_2 &= \frac{1}{2e_1^2} \\
\beta_2 &= -\frac{1}{4e_1} \\
\alpha_3 &= \frac{1}{3e_1^3} \\
\beta_3 &= -\frac{1}{8e_1^2}.
\end{aligned} \tag{5}$$

(Again, terms having eccentricity as a multiplier have been neglected).

Thus, Equations (4) become

$$e = e_1 - \left(Q - \frac{Q^3}{8e_1^2}\right)\cos\overline{\theta} - \frac{Q^2}{4e_1}\cos 2\overline{\theta} - \frac{Q^3}{8e_1^2}\cos 3\overline{\theta} + \cdots$$

$$\theta = \overline{\theta} + \left(\frac{Q}{e_1} + \frac{Q^3}{4e_1^3}\right)\sin\overline{\theta} + \frac{Q^2}{2e_1^2}\sin 2\overline{\theta} + \frac{Q^3}{3e_1^3}\sin 3\overline{\theta} + \cdots. \tag{6}$$

VON ZEIPEL METHOD

Brouwer (Reference 2) and Kozai (Reference 3) have applied the von Zeipel method to obtain solutions up to certain orders of J_2. However, if one is concerned only with long-period terms in which eccentricity does not appear as a multiplier, the determining functions may be expressed in terms of Q and hence may involve any of the zonal harmonics. Using the notation of Brouwer and Kozai, those portions of the determining functions S_1^*, S_1^*, and S_3^* to be considered here are

$$\Delta S_1^* \simeq e'' \sqrt{a''}\, Q \cos g'$$

$$\Delta S_2^* \simeq \frac{\sqrt{a''}\, Q^2}{4} \sin 2g' \tag{7}$$

$$\Delta S_3^* \simeq \frac{-\sqrt{a''}\, Q^3}{24\, e''} (3 \cos g' + \cos 3g'),$$

for which the a_0 and i_0 in Q have been replaced by a'' and i'', respectively. Then,

$$g' = g'' - \frac{\partial (\Delta S_1^*)}{\partial G''} - \frac{\partial (\Delta S_2^*)}{\partial G''} - \frac{\partial (\Delta S_3^*)}{\partial G''} \tag{8a}$$

$$G' = G'' + \frac{\partial (\Delta S_1^*)}{\partial g'} + \frac{\partial (\Delta S_2^*)}{\partial g'} + \frac{\partial (\Delta S_3^*)}{\partial g'} \tag{8b}$$

Expansion by means of Taylor's series to obtain all terms through order Q^3 yields, for Equation (8a),

$$g' = g'' - \left.\frac{\partial(\Delta S_1^*)}{\partial G''}\right|_{g'=g''} - \left.\frac{\partial(\Delta S_2^*)}{\partial G''}\right|_{g'=g''} - \left.\frac{\partial(\Delta S_3^*)}{\partial G''}\right|_{g'=g''}$$

$$+ \frac{\partial^2(\Delta S_1^*)}{\partial G''\partial g''}\left[\frac{\partial(\Delta S_1^*)}{\partial G''} + \frac{\partial(\Delta S_2^*)}{\partial G''}\right] + \frac{\partial^2(\Delta S_2^*)}{\partial G''\partial g''}\,\frac{\partial(\Delta S_1^*)}{\partial G''} - \frac{1}{2}\,\frac{\partial^3(\Delta S_1^*)}{\partial G''\partial g''^2}\left[\frac{\partial(\Delta S_1^*)}{\partial G''}\right]^2 \tag{9}$$

where

$$\frac{\partial(\Delta S_1^*)}{\partial G''} \simeq -\frac{Q}{e''}\cos g'$$

$$\frac{\partial(\Delta S_2^*)}{\partial G''} \simeq 0$$

$$\frac{\partial(\Delta S_3^*)}{\partial G''} \simeq -\frac{Q^3}{24\,e''^3}\,(3\cos g' + \cos 3g').$$

In addition,

$$\cos g' \simeq \cos\left[g'' + \frac{Q}{e''}\cos g''\right] \simeq \cos g'' - \frac{Q}{2\,e''}\sin 2g''.$$

Then, Equation (9) becomes

$$g' = g'' + \frac{Q}{e''}\cos g'' - \frac{Q^2}{2\,e''^2}\sin 2g'' - \frac{Q^3}{3\,e''^3}\cos 3g''. \tag{10}$$

However, a long-period term involving J_2^2 appears in the expression for g after the transformation from osculating elements to primed elements is performed:

$$g \simeq g' + \varphi(L', G', H', g'),$$

where

$$\varphi \simeq - \frac{9 J_2^2 \sin^2 i' (1 - 3 \cos^2 i')}{32 a'^4 e'^2} \sin 2g'.$$

But

$$\varphi(L', G', H', g') = \varphi(L'', G'', H'', g'') + \frac{\partial \varphi}{\partial G''} \frac{\partial (\Delta S_1^*)}{\partial g'} - \frac{\partial \varphi}{\partial g''} \frac{\partial (\Delta S_1^*)}{\partial G''},$$

and

$$\frac{\partial \varphi}{\partial G''} \simeq - \frac{9 J_2^2 \sin^2 i'' (1 - 3 \cos^2 i'')}{16 e''^4 a''^4 \sqrt{a''}} \sin 2g''$$

$$\frac{\partial \varphi}{\partial g''} \simeq - \frac{9 J_2^2 \sin^2 i'' (1 - 3 \cos^2 i'')}{16 e''^2 a''^4} \cos 2g''$$

$$\frac{\partial (\Delta S_1^*)}{\partial g'} \simeq - e'' \sqrt{a''}\, Q \sin g''.$$

Hence,

$$g \simeq g' - \frac{9 J_2^2 \sin^2 i'' (1 - 3 \cos^2 i'')}{32 a''^4 e''^2} \sin 2g'' - \frac{9 J_2^2 \sin^2 i'' (1 - 3 \cos^2 i'') Q}{16 a''^4 e''^3} \cos 3g''.$$

Therefore,

$$g = g'' + \frac{Q}{e''} \cos g'' - \left[\frac{Q^2}{2 e''^2} + \frac{9 J_2^2 \sin^2 i'' (1 - 3 \cos^2 i'')}{32 a''^4 e''^2} \right] \sin 2 g''$$

$$- \left[\frac{Q^3}{3 e''^3} + \frac{9 J_2^2 \sin^2 i'' (1 - 3 \cos^2 i'')}{16 a''^4 e''^3} Q \right] \cos 3g''. \quad (11)$$

Returning to Equation (8b), Taylor's Series expansions give

$$G' = G'' + \frac{\partial (\Delta S_1^*)}{\partial g''} + \frac{\partial (\Delta S_2^*)}{\partial g''} + \frac{\partial (\Delta S_3^*)}{\partial g''} - \frac{\partial^2 (\Delta S_1^*)}{\partial g''^2} \left[\frac{\partial (\Delta S_1^*)}{\partial G''} + \frac{\partial (\Delta S_2^*)}{\partial G''} \right]$$

$$- \frac{\partial^2 (\Delta S_2^*)}{\partial g''^2} \frac{\partial (\Delta S_1^*)}{\partial G''} + \frac{1}{2} \frac{\partial^3 (\Delta S_1^*)}{\partial g''^3} \left[\frac{\partial (\Delta S_1^*)}{\partial G''} \right]^2, \quad (12)$$

which becomes

$$G' = G'' - \frac{\sqrt{a''}\, Q^2}{2} - \sqrt{a''}\, e''\, Q \sin g''$$

$$= G'' + \Delta G'. \quad (13)$$

Then,

$$(\Delta G')^2 = \frac{a''\, e''^2\, Q^2}{2} + a''\, e''\, Q^3 \sin g'' - \frac{a''\, e''^2\, Q^2}{2} \cos 2 g''$$

$$(\Delta G')^3 = -\frac{3\, a'' \sqrt{a''}\, e''^3\, Q^3}{4} \sin g'' + \frac{a'' \sqrt{a''}\, e''^3\, Q^3}{4} \sin 3\, g''.$$

Since

$$\Delta e' \simeq -\frac{\Delta G'}{\sqrt{a''}\, e''} - \frac{(\Delta G')^2}{2\, a''\, e''^3} - \frac{(\Delta G')^3}{2\, a'' \sqrt{a''}\, e''^5},$$

the result is

$$e' = e'' + \frac{Q^2}{4\, e''} + \left(Q - \frac{Q^3}{8 e''^2}\right) \sin g'' + \frac{Q^2}{4 e''} \cos 2\, g'' - \frac{Q^3}{8 e''^2} \sin 3\, g''. \tag{14}$$

With

$$\theta = g + \frac{\pi}{2}$$

$$\overline{\theta} = g'' + \frac{\pi}{2}$$

$$e_1 \simeq e'' \left(1 + \frac{Q^2}{4 e''^2}\right),$$

$$i_0 = i''$$

$$a_0 = a'',$$

Equations (11) and (14) become

$$e = e_1 - \left(Q - \frac{Q^3}{8e_1^2} \right) \cos \bar{\theta} - \frac{Q^2}{4e_1} \cos 2\bar{\theta} - \frac{Q^3}{8e_1^2} \cos 3\bar{\theta}$$

$$g = g'' + \left(\frac{Q}{e_1} + \frac{Q^3}{4e_1^3} \right) \sin \bar{\theta} + \left[\frac{Q^2}{2e_1^2} + \frac{9 J_2^2 \sin^2 i_0 \, (1 - 3\cos^2 i_0)}{32\, a_0^4\, e_1^2} \right] \sin 2\bar{\theta}$$

$$+ \left[\frac{Q^3}{3e_1^3} + \frac{9 J_2^2 \sin^2 i_0 \, (1 - 3\cos^2 i_0)}{16\, a_0^4\, e_1^3} Q \right] \sin 3\bar{\theta}, \quad (15)$$

which agree with the results obtained in the Delaunay solution (Equations (6)), insofar as terms involving only Q are concerned. Equations (15) will be used in the study of Alouette 1 and Tiros 8.

ANALYSIS OF ALOUETTE 1 AND TIROS 8

Values of e'' and g'' for the two satellites, as determined by the use of the formulas in Reference 2, were first corrected for lunar and solar effects by the perturbation formulas appearing in Reference 4. These corrected values, labeled e_c'' and g_c'', appear in Tables 1, 2, 3, and 4 in Appendix B. The appearance of three near-resonant terms in the luni-solar perturbations for Alouette 1 posed a minor problem; however, they were found to not affect the eccentricity inasmuch as the argument of perigee did not appear in any of the arguments. Furthermore, since the periods of the three arguments were on the order of 50,000, 100,000, and 100,000 days, the effects on g were assumed to be manifested as a constant perturbation over the time inverval studied, and were therefore considered unimportant to the analysis.

The e_c'' and g_c'' values were then fit, by means of least squares, to the following models:

$$e_c'' = e_0 + A_1 \cos \bar{\theta} + A_2 \cos 2\bar{\theta} + \cdots + A_9 \cos 9\bar{\theta}$$

$$g_c'' = g_0 + \dot{g}_c'' (t - t_0) + B_1 \sin \bar{\theta} + B_2 \sin 2\bar{\theta} + \cdots + B_9 \sin 9\bar{\theta},$$

where

$$\overline{\theta} = \overline{\theta}_0 + \dot{\overline{\theta}}\,(t - t_0).$$

Values for $\dot{\overline{\theta}}$ were computed from Equation (2), using Kozai's (see Reference 5) determinations of J_2 and J_4, and the following values for a_0 and i_0:

Alouette 1	Tiros 8
$a_0 = 1.1589$ earth radii	$a_0 = 1.1140$ earth radii
$i_0 = 80^{\circ}.466$	$i_0 = 58^{\circ}.500$
$J_2 = 1082.645 \times 10^{-6}$	$J_4 = -1.649 \times 10^{-6}$

The results were:

Alouette 1

$$\overline{\theta} = 109^{\circ}.13743 - (2^{\circ}.5649585/\text{day})\,(t - t_0)$$

$$\begin{aligned} e_c'' = \; & .0025163652 - .0001492876 \cos\overline{\theta} - .0001336935 \cos 2\overline{\theta} - .0000097969 \cos 3\overline{\theta} \\ & - .0000264826 \cos 4\overline{\theta} + .0000007387 \cos 5\overline{\theta} - .0000042243 \cos 6\overline{\theta} \\ & - .0000082012 \cos 7\overline{\theta} - .0000070067 \cos 8\overline{\theta} - .0000001323 \cos 9\overline{\theta} \end{aligned}$$

$$\begin{aligned} g_c'' = \; & 17^{\circ}.74620 - (2^{\circ}.5618750/\text{day})\,(t - t_0) + 4^{\circ}.2076854 \sin\overline{\theta} + 4^{\circ}.6340045 \sin 2\overline{\theta} \\ & + 0^{\circ}.1873826 \sin 3\overline{\theta} + 0^{\circ}.7003276 \sin 4\overline{\theta} - 0^{\circ}.0066516 \sin 5\overline{\theta} \\ & + 0^{\circ}.1206393 \sin 6\overline{\theta} + 0^{\circ}.0102137 \sin 7\overline{\theta} + 0^{\circ}.0387951 \sin 8\overline{\theta} - 0^{\circ}.0680971 \sin 9\overline{\theta} \end{aligned}$$

Tiros 8

$$\overline{\theta} = 213^{\circ}.61150 + (1^{\circ}.2452865/\text{day})\,(t - t_0)$$

$$e_c'' = .0034394605 - .0004525939 \cos \overline{\theta} - .0001389608 \cos 2\overline{\theta}$$

$$- .0000164065 \cos 3\overline{\theta} - .0000155041 \cos 4\overline{\theta} - .0000064148 \cos 5\overline{\theta}$$

$$- .0000043222 \cos 6\overline{\theta} - .0000027052 \cos 7\overline{\theta} - .0000029382 \cos 8\overline{\theta}$$

$$+ .0000069836 \cos 9\overline{\theta}$$

$$g_c'' = 125^\circ\!.65579 + (1^\circ\!.2412695/\text{day})\,(t - t_0) + 8^\circ\!.0967578 \sin \overline{\theta}$$

$$+ 3^\circ\!.5255978 \sin 2\overline{\theta} + 0^\circ\!.5354116 \sin 3\overline{\theta} + 0^\circ\!.4133096 \sin 4\overline{\theta}$$

$$+ 0^\circ\!.1799011 \sin 5\overline{\theta} + 0^\circ\!.1123452 \sin 6\overline{\theta} + 0^\circ\!.1127257 \sin 7\overline{\theta}$$

$$+ 0^\circ\!.1213947 \sin 8\overline{\theta} + 0^\circ\!.0427651 \sin 9\overline{\theta}$$

Values of

$$e_0 + A_1 \cos \overline{\theta} + A_2 \cos 2\overline{\theta} + \cdots + A_9 \cos 9\overline{\theta},$$

$$g_c'' - [g_0 + \dot{g}_c''(t - t_0)],$$

and

$$B_1 \sin \overline{\theta} + B_2 \sin 2\overline{\theta} + \cdots + B_9 \sin 9\overline{\theta}$$

for the two satellites are also listed in the tables in Appendix B. The graphs in Appendix C show the closeness of the fits.

In order to compare the least squares results with Equations (15), corrections must be applied to the observed amplitudes of the $\sin \overline{\theta}$ and $\cos \overline{\theta}$ terms, in view of the fact that J_3 and J_5 (to the first power) have been accounted for in the computation of e'' and g''. The values used were

$$J_3 = -2.285 \times 10^{-6}, \quad J_5 = -0.232 \times 10^{-6}.$$

Using these numbers, the values previously given for a_0, i_0, J_2, and J_4, the eccentricity constants in the least squares results, and Equations (15), one finds that the following terms must be added to the series determined by least squares:

Alouette 1	Tiros 8
$\Delta e_c'' = -.9626 \times 10^{-3} \cos \bar{\theta}$	$\Delta e_c'' = -1.0993 \times 10^{-3} \cos \bar{\theta}$
$\Delta g_c'' = 21^\circ.920 \sin \bar{\theta}$	$\Delta g_c'' = 18^\circ.312 \sin \bar{\theta}$

Therefore, the following series are the ones to be compared with the trigonometric terms in Equations (15):

Alouette 1

$$\Delta e_c'' = -.0011119 \cos \bar{\theta} - .00013369 \cos 2\bar{\theta} - .0000097969 \cos 3\bar{\theta}$$

$$\Delta g_c'' = 0.45602 \sin \bar{\theta} + 0.080879 \sin 2\bar{\theta} + 0.0032704 \sin 3\bar{\theta} \text{ (radians)} \tag{16}$$

Tiros 8

$$\Delta e_c'' = -.0015519 \cos \bar{\theta} - .00013896 \cos 2\bar{\theta} - .000016406 \cos 3\bar{\theta}$$

$$\Delta g_c'' = 0.46092 \sin \bar{\theta} + 0.061533 \sin 2\bar{\theta} + 0.0093447 \sin 3\bar{\theta} \text{ (radians)} \tag{17}$$

Examination of Equations (15) shows that values for Q and e_1 may be computed from the observed amplitudes of the $\sin \bar{\theta}$ and $\cos \bar{\theta}$ terms, i.e.,

Alouette 1

$$\frac{Q}{e_1} + \frac{Q^3}{4e_1^3} = 0.45602 \Rightarrow Q = 0.43539\, e_1$$

$$Q - \frac{Q^3}{8e_1^2} = 0.0011119, \; Q = 0.43539\, e_1 \Rightarrow e_1 = 0.0026158, \; Q = 0.0011389$$

Tiros 8

$$\frac{Q}{e_1} + \frac{Q^3}{4e_1^3} = 0.46092 \Rightarrow Q = 0.43967\, e_1$$

$$Q - \frac{Q^3}{8e_1^2} = 0.0015519, \quad Q = 0.43967\, e_1 \Rightarrow e_1 = 0.0036170, \quad Q = 0.0015903$$

Using these values of e_1 and Q, predicted amplitudes for the $2\bar{\theta}$ and $3\bar{\theta}$ terms can be computed (referring to Equations (15)):

Alouette 1

$$\cos 2\bar{\theta}: -\frac{Q^2}{4e_1} = -0.00012397 \quad (\text{observed:} \quad -0.00013369)$$

$$\cos 3\bar{\theta}: -\frac{Q^3}{8e_1^2} = -0.000026987 \quad (\text{observed:} \quad -0.0000097969)$$

$$\sin 2\bar{\theta}: \quad \frac{Q^2}{2e_1^2} + \frac{9J_2^2 \sin^2 i_0 (1 - 3\cos^2 i_0)}{32\, a_0^4\, e_1^2} = 0.118625 \quad (\text{observed:} \quad 0.080879)$$

$$\sin 3\bar{\theta}: \quad \frac{Q^3}{3e_1^3} + \frac{9J_2^2 \sin^2 i_0 (1 - 3\cos^2 i_0)}{16\, a_0^4\, e_1^3}\, Q = 0.048273 \quad (\text{observed:} \quad 0.0032704)$$

Tiros 8

$$\cos 2\bar{\theta}: \quad -\frac{Q^2}{4e_1} = -0.00017480 \quad (\text{observed:} \ -0.00013896)$$

$$\cos 3\bar{\theta}: \quad -\frac{Q^3}{8e_1^2} = -0.000038428 \quad (\text{observed:} \ -0.000016406)$$

$$\sin 2\bar{\theta}: \quad \frac{Q^2}{2e_1^2} + \frac{9J_2^2 \sin^2 i_0 (1 - 3\cos^2 i_0)}{32\, a_0^4\, e_1^2} = 0.098809 \quad (\text{observed:} \ 0.061533)$$

$$\sin 3\bar{\theta}: \quad \frac{Q^3}{3e_1^3} + \frac{9J_2^2 \sin^2 i_0 (1 - 3\cos^2 i_0)}{16\, a_0^4\, e_1^3}\, Q = 0.030225 \quad (\text{observed:} \ 0.0093447)$$

For the most part, these predicted amplitudes are of the same order of magnitude as the observed ones – scatter in the data probably prevents better agreement.

From Equations (2) and (3), one finds that

Alouette 1: $Q = -(393.76\, J_3 + 268.41\, J_5 + 114.96\, J_7 - 0.82035\, J_9 - 70.080\, J_{11})$

Tiros 8: $Q = -(355.56\, J_3 + 1233.2\, J_5 + 638.42\, J_7 - 434.58\, J_9 - 771.97\, J_{11})$.

Using the eccentricity constants from the least squares results, the relation

$$e_1 \simeq e'' \left(1 + \frac{Q^2}{4\, e''^2}\right),$$

and Kozai's (Reference 5) values for the zonal harmonics:

$$J_3 = -2.546 \times 10^{-6},\ J_5 = -0.210 \times 10^{-6},\ J_7 = -0.333 \times 10^{-6}$$

$$J_9 = -0.053 \times 10^{-6},\ J_{11} = 0.302 \times 10^{-6},$$

one obtains

Alouette 1: $Q = 0.0011183,\ e_1 = 0.0026406$

Tiros 8: $Q = 0.0015869,\ e_1 = 0.0036225$

These numbers are in quite good agreement with those derived from the data; if harmonics beyond J_{11}, had been used, the theoretical values of Q would have been even closer to the observed values.

It is interesting to note that the eccentricity constant e_1 used here is essentially the e_0 of Kozai (Reference 3).

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APPENDIX A

Symbols

a = semi-major axis of satellite's orbit

e = eccentricity of satellite's orbit

i = inclination of satellite's orbital plane to earth's equatorial plane

g = argument of perigee of satellite

$\theta = g + \frac{\pi}{2}$

$L', G', H', L'', G'', H'', g', g''$ = Delaunay variables, used in von Zeipel method

μ = product of gravitational constant with the mass of the earth

$a' = \mu^{-1} L'^2$

$a'' = \mu^{-1} L''^2$

$e' = \sqrt{1 - G'^2/L'^2}$

$e'' = \sqrt{1 - G''^2/L''^2}$

$i' = \cos^{-1}\left(\frac{H'}{G'}\right)$

$i'' = \cos^{-1}\left(\frac{H''}{G''}\right)$

a_0 = semi-major axis constant

e_0, e_1 = eccentricity constants

i_0 = inclination constant

g_0 = argument of perigee constant

e_c'' = e'' , less luni-solar gravitational perturbations

g_c'' = g'' , less luni-solar gravitational perturbations

$t - t_0$ = time elapsed since initial epoch

$\overline{\theta}$ = constant + $\dot{\overline{\theta}}$ $(t - t_0) \simeq g'' + \pi/2$

$\dot{\overline{\theta}}$ = mean motion of $\overline{\theta}$ = mean motion of g''

$\dot{g}_c''$ = mean motion of g_c''

$J_2, J_3, J_4, J_5, J_7, J_9, J_{11}$ = zonal harmonic coefficients in earth's gravitational potential

S_1^*, S_2^*, S_3^* = long-period determining functions, used in von Zeipel method

APPENDIX B

Tables

TABLE 1. Eccentricity of Alouette 1

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\tilde{\theta}\right) \times 10^2$
0	.26144376	.26537030
7	.25871777	.26220757
14	.25607286	.25939988
21	.24449395	.24958975
28	.23735904	.23887395
35	.22244876	.22744022
49	.22755925	.22533918
56	.24002016	.23735173
70	.26071055	.25837643
77	.26551042	.26191449
84	.26744275	.26482671
98	.25913103	.26195223
119	.25383359	.25431977
126	.25626008	.26072945
133	.26025462	.26565796
140	.26297148	.26553041
147	.25871509	.26231250
154	.26134549	.25967895
161	.24748761	.25019792
168	.24020682	.23935735
182	.21334841	.21800599
189	.22432010	.22467505
196	.23247997	.23685073
201	.24577516	.24391138
208	.26312523	.25540848
215	.26812333	.26129415
222	.26386234	.26360335
236	.26792922	.26376418
243	.25908024	.25783860
250	.24963096	.25201272
257	.25192732	.25250961
264	.25645127	.25862181
271	.26108098	.26434214
278	.26628269	.26629414
285	.25685775	.26320387
292	.25121652	.26099828
299	.25179609	.25412753

TABLE 1. Eccentricity of Alouette 1 (Continued)

$t - t_0$ (days)	$e''_c \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\bar{\theta}\right) \times 10^2$
306	.23714491	.24265445
313	.22717008	.23241403
320	.22112633	.22003809
327	.22437962	.22070388
334	.23871339	.23323816
341	.25044991	.24337748
348	.25785615	.25488266
362	.27062429	.26343142
369	.27130551	.26636918
376	.26925108	.26401216
383	.26114939	.25816855
390	.25307810	.25220931
397	.25245587	.25228939
404	.25637716	.25829464
411	.26291397	.26410523
418	.26170294	.26635163
425	.26389721	.26336706
432	.25789832	.26113067
439	.25016203	.25467635
446	.24019515	.24317523
455	.22589934	.22945224
462	.21591834	.21838892
469	.22068942	.22338346
476	.23533547	.23582200
483	.24822932	.24595138
489.70486	.25030181	.25678984
496.70486	.26066031	.26157692
503.70486	.26765384	.26411447
510.70486	.27155964	.26640223
517.70486	.26437484	.26302577
524.70486	.25939428	.25689890
531.70486	.25239343	.25155922
538.70486	.25124586	.25321344
545.70486	.25564045	.25953301
552.70486	.26061170	.26496173
557.70486	.26295045	.26641240
564.70486	.26095131	.26368288

TABLE 1. Eccentricity of Alouette 1 (Continued)

$t - t_0$ (days)	$e''_c \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\bar{\theta}\right) \times 10^2$
571.70486	.25934972	.26134336
578.70486	.25650770	.25564022
584.70486	.24500476	.24573405
591.70486	.23457316	.23562205
598.70486	.22485878	.22314570
605.70486	.22265337	.21848485
612.70486	.23192097	.22970239
619.70486	.24308295	.24051097
626.70486	.25162352	.25163740
633.70486	.26184213	.26025111
640.70486	.26530320	.26259063
647.70486	.27353441	.26587053
654.70486	.27184750	.26524760
661.70486	.26447327	.25999302
667.70486	.25222628	.25445884
674.70486	.25103885	.25119352
681.70486	.25335351	.25548038
688.70486	.25820461	.26182112
695.70486	.26029715	.26613031
702.70486	.26490152	.26490888
709.70486	.26249207	.26195477
716.70486	.25713385	.25853996
723.70486	.24794743	.24798143
730.70486	.23423279	.23757589
737.70486	.22400092	.22564218
744.70486	.21910726	.21786121
751.70486	.23313638	.22713393
758.70486	.24311689	.23865496
765.70486	.24894646	.24931496
772.70486	.26140113	.25926586
779.70486	.25985893	.26216213
786.70489	.26040271	.26529515
794.70486	.27028544	.26542908
804.70486	.26504396	.25756433
808.70486	.25435655	.25390550
822.70486	.25505307	.25608246
829.70486	.25681805	.26234663

TABLE 1. Eccentricity of Alouette 1 (Continued)

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\overline{\theta}\right) \times 10^2$
836.70486	.26084080	.26628561
843.70486	.25794681	.26457214
850.70486	.25903076	.26179389
857.70486	.26195972	.25784642
884.70486	.21750902	.21782818
891.70486	.22434360	.22645646
898.70486	.24179434	.23816861
905.70486	.24720073	.24870863
911.70486	.26352571	.25792895
918.70486	.26893127	.26181188
925.70486	.27281346	.26461062
932.70486	.27063842	.26627094
940.70486	.26949408	.26146684
954.70486	.25533530	.25116558
961.70486	.25193124	.25483685
975.70486	.26561079	.26589761
982.70486	.26585615	.26526020
996.70486	.25891672	.25920282

$e_0 = .0025163652$ $\quad A_1 = -.0001492876$ $\quad A_2 = -.0001336935$

$A_3 = -.0000097969$ $\quad A_4 = -.0000264826$ $\quad A_5 = .0000007387$

$A_6 = -.0000042243$ $\quad A_7 = -.0000082012$ $\quad A_8 = -.0000070067$

$A_9 = -.0000001323$ $\quad \overline{\theta} = 109^\circ.13743 - (2^\circ.5649585/\text{day})\,(t - t_0)$

TABLE 2. Argument of Perigee of Alouette 1

$t - t_0$ (days)	g_c'' (deg.)	$g_c'' - [g_0 + \dot{g}_c'' (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\overline{\theta}$ (deg.)
0	1099.31099	1.56480	1.66824
7	1084.38172	4.56862	3.79416
14	1067.61000	5.73005	5.97897
21	1050.38279	6.43597	7.32940
28	1032.26639	6.25270	7.45176
35	1013.54196	5.46142	5.25543
49	967.47511	-4.73922	-4.62688
56	947.84411	-6.43709	-7.30885
70	911.67829	-6.73666	-6.27662
77	896.39081	-4.09102	-4.13321
84	880.80939	-1.73931	-2.02219
98	848.42732	1.74487	1.94181
119	791.28301	-1.60007	-1.98001
126	773.83660	-1.11335	-2.11188
133	757.18526	.16843	- .77541
140	740.11901	1.03531	1.55155
147	724.76300	3.61242	3.68696
154	707.99420	4.77675	5.87760
161	692.34621	7.06188	7.29129
168	674.36622	7.01502	7.48468
182	627.80422	-3.68073	.69798
189	609.52750	-4.02433	-4.41088
196	588.52382	-7.09488	-7.24896
201	575.71039	-7.09894	-7.55947
208	558.96521	-5.91099	-6.80886
215	542.91323	-4.02985	-4.88339
222	526.50266	-2.50729	-2.73991
236	496.45747	3.31427	1.55125
243	479.05674	3.84616	2.28482
250	459.97451	2.69706	1.15849
257	439.03768	- .30665	-1.42359
264	420.29035	-1.12085	-2.26965
271	403.05640	- .42168	-1.37480
278	386.63545	1.09050	.74730
285	370.72010	3.10827	2.98848
292	354.69434	5.01564	5.15365

TABLE 2. Argument of Perigee of Alouette 1 (Continued)

$t - t_0$ (days)	g_c'' (deg.)	$g_c'' - [g_0 + \dot{g}_c'' (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\bar{\theta}$ (deg.)
299	338.16865	6.42307	6.96186
306	320.44346	6.63101	7.57280
313	300.43327	4.55394	6.48325
320	278.97242	1.02622	2.47439
327	254.45608	-5.55699	-2.82822
334	233.25165	-8.82830	-6.65280
341	215.49481	-8.65202	-7.56748
348	198.67583	-7.53787	-6.87546
362	167.44797	-2.89948	-2.84445
369	152.10532	- .30901	- .57744
376	136.67553	2.19433	1.47965
383	119.89750	3.34942	2.28144
390	100.44572	1.83077	1.27344
397	80.31454	- .36729	-1.31618
404	61.59278	-1.15592	-2.27901
411	44.11512	- .70046	-1.45130
418	27.74386	.86141	.62450
425	12.30852	3.35919	2.88440
432	- 3.65805	5.32575	5.04053
439	- 19.80376	7.11316	6.90004
446	- 38.01440	6.83565	7.56960
455	- 62.10665	5.80027	5.79301
462	- 85.22013	.61992	1.24025
469	-109.03395	-5.26078	-3.96046
476	-129.72493	-8.01863	-7.10704
483	-147.85210	-8.21268	-7.50396
489.70486	-163.46894	-6.65250	-6.60175
496.70486	-179.87633	-5.12677	-4.56244
503.70486	-195.74486	-3.06199	-2.44097
510.70486	-210.88356	- .26775	- .11219
517.70486	-225.88114	2.66780	1.73491
524.70486	-243.75366	2.72840	2.26852
531.70486	-262.58594	1.82925	.80781
538.70486	-283.03946	- .69115	-1.69490
545.70486	-301.55471	-1.27327	-2.22192
552.70486	-318.52399	- .30943	-1.13936

TABLE 2. Argument of Perigee of Alouette 1 (Continued)

$t - t_0$ (days)	g_c'' (deg.)	$g_c'' - [g_0 + \dot{g}_c'' (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\overline{\theta}$ (deg.)
557.70486	-330.59113	.43281	.39964
564.70486	-346.42503	2.53203	2.69249
571.70486	-362.28373	4.60646	4.83200
578.70486	-377.44955	7.37376	6.77763
584.70486	-392.75477	7.43979	7.51143
591.70486	-410.97344	7.15425	7.07658
598.70486	-434.21879	1.84202	3.87229
605.70486	-455.35533	-1.36139	-1.34258
612.70486	-478.35659	-6.42953	-5.85601
619.70486	-498.33821	-8.47802	-7.54058
626.70486	-515.50363	-7.71032	-7.18929
633.70486	-531.34169	-5.61525	-5.62738
640.70486	-546.94966	-3.29010	-3.43513
647.70486	-562.43236	- .83967	-1.26893
654.70486	-576.75933	2.76648	1.00699
661.70486	-594.06069	3.39825	2.18588
667.70486	-609.07338	3.75681	1.99732
674.70486	-629.97554	.78777	- .22544
681.70486	-649.07326	- .37682	-2.16137
688.70486	-667.32226	- .69270	-1.96313
695.70486	-683.65836	.90433	- .38545
702.70486	-699.55967	2.93614	1.97156
709.70486	-715.70662	4.72232	4.08336
716.70486	-732.30086	6.06120	6.23510
723.70486	-750.52649	5.76870	7.41737
730.70486	-768.25612	5.97219	7.33361
737.70486	-788.66003	3.50141	4.72236
744.70486	-811.32247	-1.22791	- .30179
751.70486	-834.31196	-6.28427	-5.16844
758.70486	-853.71462	-7.75381	-7.43487
765.70486	-872.67092	-8.77692	-7.34571
772.70486	-888.45325	-6.62619	-6.02391
779.70486	-903.40289	-3.64270	-3.84290
786.70486	-920.41593	-2.72262	-1.72051
794.70486	-936.98729	1.20102	.91193

TABLE 2. Argument of Perigee of Alouette 1 (Continued)

$t - t_0$ (days)	g''_c (deg.)	$g''_c - [g_0 + \dot{g}''_c (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\bar{\theta}$ (deg.)
804.70486	- 959.72034	4.08672	2.28405
808.70486	- 971.13202	2.92254	1.88870
822.70486	-1010.85152	- .93069	-2.22030
829.70486	-1029.19917	-1.34526	-1.87294
836.70486	-1046.28979	- .50274	- .17727
843.70486	-1062.14499	1.57519	2.17480
850.70486	-1078.27671	3.37661	4.28622
857.70486	-1094.47075	5.11564	6.39915
884.70486	-1168.06836	.68866	- .02841
891.70486	-1192.05474	-5.36451	-4.97071
898.70486	-1213.49333	-8.86999	-7.39306
905.70486	-1231.15939	-8.60296	-7.37985
911.70486	-1245.52583	-7.59811	-6.38134
918.70486	-1260.73638	-4.87559	-4.26335
925.70486	-1275.36111	-1.56716	-2.15224
932.70486	-1291.62282	.10426	.20143
940.70486	-1309.48614	2.73596	2.01669
954.70486	-1345.35347	2.73481	.05570
961.70486	-1367.39337	-1.37196	-2.07316
975.70486	-1402.75694	- .86921	- .60340
982.70486	-1419.34065	.48011	1.74421

$g_0 = 1097.74620$ $\dot{g}''_c = -2^{\circ}.5618750/\text{day}$ $B_1 = 4^{\circ}.2076854$ $B_2 = 4^{\circ}.6340045$

$B_3 = 0^{\circ}.1873826$ $B_4 = 0^{\circ}.7003276$ $B_5 = 0^{\circ}.0066516$ $B_6 = 0^{\circ}.1206393$

$B_7 = 0^{\circ}.0102137$ $B_8 = 0^{\circ}.0387951$ $B_9 = -0^{\circ}.0680971$

$\bar{\theta} = 109^{\circ}.13743 - (2^{\circ}.5649585/\text{day})(t - t_0)$

TABLE 3. Eccentricity of Tiros 8

$t - t_0$ (days)	$e''_c \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\bar{\theta}\right) \times 10^2$
0	.37506961	.37628641
4	.37619047	.37570213
10.99306	.37460858	.37520643
17.99306	.37238024	.37446838
24.99306	.37085765	.37171128
31.99306	.36733611	.36676298
38.99306	.36039354	.36119998
45.99306	.35743823	.35587962
52.99306	.35615808	.34972838
59.99306	.33757149	.34161870
66.99306	.33314402	.33233538
73.99306	.31740621	.32357566
80.99306	.31274407	.31560048
87.99306	.30286917	.30718518
94.99306	.29095610	.29789457
101.99306	.28879741	.28924569
108.99306	.28080994	.28324962
115.99306	.27858500	.28074478
122.99306	.27612262	.28169755
129.99306	.28875311	.28616007
136.99306	.29812405	.29382323
143.99306	.30691754	.30309784
150.99306	.30825442	.31197276
157.99306	.31510139	.31998188
164.99306	.33099291	.32829756
171.99306	.33561471	.33751327
177.99306	.33987896	.34518655
183.99306	.35051776	.35155119
190.99306	.35399237	.35731930
196.99306	.36010977	.36188160
203.99306	.36691971	.36745437
210.99306	.36883817	.37219755
217.99306	.37460394	.37464193
224.99306	.37351657	.37524503
231.99306	.37470128	.37581423
238.99306	.37580827	.37690387
245.99306	.37346974	.37725685
252.99306	.37382165	.37616075
259.99306	.37218264	.37497208

TABLE 3. Eccentricity of Tiros 8 (Continued)

$t - t_0$ (days)	$e''_c \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\bar{\theta}\right) \times 10^2$
266.99306	.37528776	.37532137
273.99306	.37495581	.37672327
280.99306	.37767549	.37729502
287.99306	.37789616	.37646770
294.99306	.37447423	.37550382
301.99306	.37530401	.37510934
308.99306	.37283971	.37396844
315.99306	.37287115	.37052801
322.99306	.36830645	.36523473
329.99306	.36135782	.35974035
336.99306	.35292034	.35435431
343.99306	.34782399	.34771606
350.99306	.34033867	.33912574
357.99306	.33043465	.32984273
364.99306	.32312787	.32136150
378.99306	.30475120	.30470470
385.99306	.29380197	.29537618
392.99306	.28598036	.28728450
396.99306	.28557000	.28395876
414.99306	.28209103	.28312168
421.99306	.28255986	.28901708
428.99306	.29979616	.29761177
435.58889	.30805258	.30639272
442.58889	.31621209	.31489632
449.58889	.31967711	.32285454
456.58889	.33049635	.33152425
463.58889	.34108957	.34082264
470.58889	.34535766	.34909678
477.58889	.35227630	.35539594
484.58889	.35844634	.36072574
491.58889	.36343811	.36627238
498.58889	.37040260	.37134748
505.58889	.37353149	.37432617
512.58889	.37415101	.37517826
519.58889	.37646365	.37563213
526.58889	.37427000	.37667850
533.58889	.37487808	.37731041
540.58889	.37637825	.37647366

TABLE 3. Eccentricity of Tiros 8 (Continued)

$t - t_0$ (days)	$e''_c \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\bar{\theta}\right) \times 10^2$
547.58889	.37514961	.37512945
554.58889	.37589851	.37510403
561.58889	.38017558	.37643276
568.58889	.37948983	.37730770
575.58889	.37910182	.37671023
582.58889	.37713322	.37565452
589.58889	.37896258	.37518764
595.58889	.37454717	.37458753
603.58889	.37299622	.37146791
610.58889	.37132891	.36643251
617.58889	.36500245	.36087979
624.58889	.35629898	.35555421
631.58889	.35118739	.34930429
638.58889	.34392614	.34108286
645.58889	.33500286	.33178773
652.58889	.32298911	.32308859
659.58889	.31421141	.31512612
666.58889	.30859248	.30665164
673.58889	.29948474	.29734106
680.58889	.28822450	.28880022
687.58889	.28074034	.28300203
694.58889	.28409691	.28070556
701.58889	.28624389	.28186334
708.58889	.28773906	.28653461
715.58889	.29714635	.29435071
722.58889	.30602984	.30365109
729.58889	.31626900	.31246574
736.58889	.31880058	.32045329
743.58889	.32718453	.32882380
750.58889	.33949012	.33806790
757.58889	.34328959	.34682916
764.58889	.35768636	.35369119
771.58889	.35838231	.35913877
778.58889	.36159859	.36459420
792.58889	.37257410	.37370595
799.58889	.37840063	.37505557
806.58889	.37739372	.37543823
813.58889	.37577636	.37633588

TABLE 3. Eccentricity of Tiros 8 (Continued)

$t - t_0$ (days)	$e''_c \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\bar{\theta}\right) \times 10^2$
820.58889	.37751534	.37725826
827.58889	.37799798	.37686126
834.58889	.37371363	.37546057
841.58889	.37525164	.37492026
848.58889	.37966114	.37599023
855.58889	.38152254	.37719711
862.58889	.37475111	.37701033
869.58889	.37627181	.37592491
876.58889	.37252805	.37528131
883.58889	.37785191	.37477231
890.58889	.37263262	.37260474
897.58889	.36647209	.36807100
904.58889	.36525288	.36250619
911.58889	.35752890	.35716509
918.58889	.35083585	.35136084
925.58889	.34323831	.34372833
932.58889	.34242113	.33456261
939.58889	.32710771	.32557333
946.58889	.32017274	.31749620
952.58889	.31498104	.31052414
959.58889	.30196193	.30148605
966.58889	.28533687	.29233065
973.58889	.28808723	.28514244
980.58889	.28253207	.28128755
987.58889	.28178518	.28092661
994.58889	.28628806	.28403749
1001.58889	.29114240	.29058966

$e_0 = .0034394605$ $A_1 = -.0004525939$ $A_2 = -.0001389608$

$A_3 = -.0000164065$ $A_4 = -.0000155041$ $A_5 = -.0000064148$

$A_6 = -.0000043222$ $A_7 = -.0000027052$ $A_8 = -.0000029382$

$A_9 = .0000069836$ $\bar{\theta} = 213^\circ\!.61150 + (1^\circ\!.2452865/\text{day})\,(t - t_0)$

TABLE 4. Argument of Perigee of Tiros 8

$t - t_0$ (days)	g''_c (deg.)	$g''_c - [g_0 + \dot{g}''_c (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\bar{\theta}$ (deg.)
0	-236.38850	-2.04429	-1.53353
4	-231.73110	-2.35197	-1.93639
10.99306	-223.82704	-3.12818	-2.73801
17.99306	-216.04005	-4.03008	-3.67624
24.99306	-208.48143	-5.16034	-4.75277
31.99306	-200.83191	-6.19971	-5.88175
38.99306	-193.33678	-7.39347	-6.90784
45.99306	-185.33747	-8.08305	-7.74952
52.99306	-177.78234	-9.21680	-8.47321
59.99306	-169.81386	-9.93721	-9.14937
66.99306	-161.14324	-9.95548	-9.66197
73.99306	-152.21856	-9.71968	-9.76938
80.99306	-143.75699	-9.94700	-9.37769
87.99306	-133.50278	-8.38168	-8.63201
94.99306	-124.28197	-7.84975	-7.63543
101.99306	-114.91644	-7.17311	-6.16535
108.99306	-105.35505	-6.30061	-3.86079
115.99306	- 89.66564	.69991	- .74903
122.99306	- 75.61633	6.06034	2.54710
129.99306	- 67.09591	5.89187	5.25347
136.99306	- 57.01023	7.28867	7.06225
143.99306	- 47.03913	8.57088	8.22353
150.99306	- 36.88613	10.03499	9.07706
164.99306	- 20.06909	9.47426	9.77187
171.99306	- 11.35478	9.49968	9.41251
177.99306	- 4.33190	9.07494	8.87943
183.99306	2.62983	8.58906	8.28431
190.99306	10.66657	7.93690	7.54004
196.99306	16.95969	6.78141	6.78911
203.99306	24.78787	5.92170	5.74262
210.99306	32.64922	5.09417	4.61202
217.99306	39.80285	3.55891	3.55044
224.99306	48.05398	3.12115	2.63039
231.99306	56.30848	2.68677	1.84372
238.99306	63.83417	1.52357	1.18418
245.99306	72.45159	1.45210	.67331

TABLE 4. Argument of Perigee of Tiros 8 (Continued)

$t - t_0$ (days)	g''_c (deg.)	$g''_c - [g_0 + \dot{g}''_c (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\bar{\theta}$ (deg.)
252.99306	80.74838	1.06000	.31552
259.99306	88.97464	.59739	.06574
266.99306	97.86244	.79630	- .15698
273.99306	105.75508	.00005	- .44144
280.99306	114.11662	- .32730	- .85767
287.99306	122.33638	- .79643	-1.43047
294.99306	130.00547	-1.81622	-2.14224
301.99306	137.77430	-2.73628	-2.97953
308.99306	145.60725	-3.59222	-3.95778
315.99306	153.02459	-4.86376	-5.06178
322.99306	161.08507	-5.49217	-6.17801
329.99306	168.64011	-6.62602	-7.15528
336.99306	176.39026	-7.56475	-7.95375
343.99306	184.02055	-8.62335	-8.66309
350.99306	192.56864	-8.76415	-9.31534
357.99306	200.29325	-9.72843	-9.74030
364.99306	209.28824	-9.42232	-9.70909
378.99306	227.47850	-8.60984	-8.38776
385.99306	237.73345	-7.04377	-7.29803
392.99306	248.87318	-4.59293	-5.62938
396.99306	254.09261	-4.33858	-4.28094
414.99306	281.67401	.89997	3.77656
421.99306	293.92654	4.46362	6.10914
428.99306	304.88601	6.73420	7.59980
435.58889	314.60638	8.26736	8.55505
442.58889	323.79059	8.76269	9.32326
449.58889	332.82721	9.11042	9.75414
456.58889	341.20804	8.80237	9.69090
463.58889	349.83079	8.73623	9.20441
470.58889	357.87348	8.09003	8.53474
477.58889	365.59878	7.12644	7.81628
484.58889	373.55167	6.39044	6.98932
491.58889	381.87638	6.02627	5.97841
498.58889	389.04468	4.50568	4.85216
505.58889	397.09801	3.87012	3.76595
512.58889	403.69348	1.77671	2.81483

TABLE 4. Argument of Perigee of Tiros 8 (Continued)

$t - t_0$ (days)	g''_c (deg.)	$g''_c - [g_0 + \dot{g}''_c (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\bar{\theta}$ (deg.)
519.58889	412.20186	1.59621	2.00134
526.58889	420.73651	1.44197	1.31337
533.58889	428.68714	.70372	.76898
540.58889	437.28478	.61247	.38057
547.58889	445.77413	.41293	.11376
554.58889	453.39280	- .65729	- .10716
561.58889	462.25789	- .48109	- .37148
568.58889	470.68661	- .74125	- .75567
575.58889	478.61534	- 1.50141	-1.29559
582.58889	486.65216	- 2.15348	-1.97982
589.58889	494.53630	- 2.95823	-2.78966
595.58889	500.92154	- 4.02060	-3.59222
603.58889	510.16508	- 4.70721	-4.81970
610.58889	517.66721	- 5.89397	-5.94698
617.58889	525.25424	- 6.99583	-6.96291
624.58889	533.15538	- 7.78358	-7.79459
631.58889	540.96597	- 7.66187	-8.51467
638.58889	548.93491	- 9.38182	-9.18656
645.58889	557.13811	- 9.86751	-9.68179
652.58889	565.22543	-10.46908	-9.75950
659.58889	575.08680	- 9.29659	-9.34122
666.58889	584.65118	- 8.42110	-8.58024
673.58889	594.21800	- 7.43117	-7.56515
680.58889	604.07334	- 6.37672	-6.05440
687.58889	616.27122	- 2.86772	-3.69494
694.58889	626.58839	- 1.23944	- .55013
701.58889	638.92826	2.41154	2.73047
708.58889	650.86627	5.66067	5.38560
715.58889	660.95223	7.05774	7.14523
722.58889	671.07217	8.48879	8.28075
729.58889	680.65811	9.38585	9.12046
736.58889	689.57999	9.61884	9.67293
743.58889	698.54485	9.89481	9.76275
750.58889	706.80434	9.46542	9.38007
757.58889	714.97396	8.94614	8.74158
764.58889	722.38052	7.66383	8.03699

TABLE 4. Argument of Perigee of Tiros 8 (Continued)

$t - t_0$ (days)	g_c'' (deg.)	$g_c'' - [g_0 + \dot{g}_c'' (t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\bar{\theta}$ (deg.)
771.58889	730.82908	7.42350	7.25423
778.58889	738.96423	6.86976	6.29846
792.58889	753.42075	3.94850	4.07828
799.58889	760.48241	2.32128	3.08342
806.58889	769.22128	2.37126	2.23113
813.58889	777.17153	1.63262	1.50499
820.58889	785.09547	.86768	.91505
827.58889	793.83962	.92294	.48126
834.58889	802.00001	.39445	.18468
841.58889	809.85732	- .43712	- .04066
848.58889	819.50980	.52646	- .28291
855.58889	828.02533	.35310	- .62523
862.58889	835.39453	- .96658	-1.11808
869.58889	843.93954	-1.11045	-1.76196

$g_0 = -234\overset{\circ}{.}34421$ $\dot{g}_c'' = 1\overset{\circ}{.}2412695/\text{day}$ $B_1 = 8\overset{\circ}{.}0967578$ $B_2 = 3\overset{\circ}{.}5255978$

$B_3 = 0\overset{\circ}{.}5354116$ $B_4 = 0\overset{\circ}{.}4133096$ $B_5 = 0\overset{\circ}{.}1799011$ $B_6 = 0\overset{\circ}{.}1123452$

$B_7 = 0\overset{\circ}{.}1127257$ $B_8 = 0\overset{\circ}{.}1213947$ $B_9 = 0\overset{\circ}{.}0427651$ $\bar{\theta} = 213\overset{\circ}{.}61150 + (1\overset{\circ}{.}2452865/\text{day})(t - t_0)$

APPENDIX C

Graphs

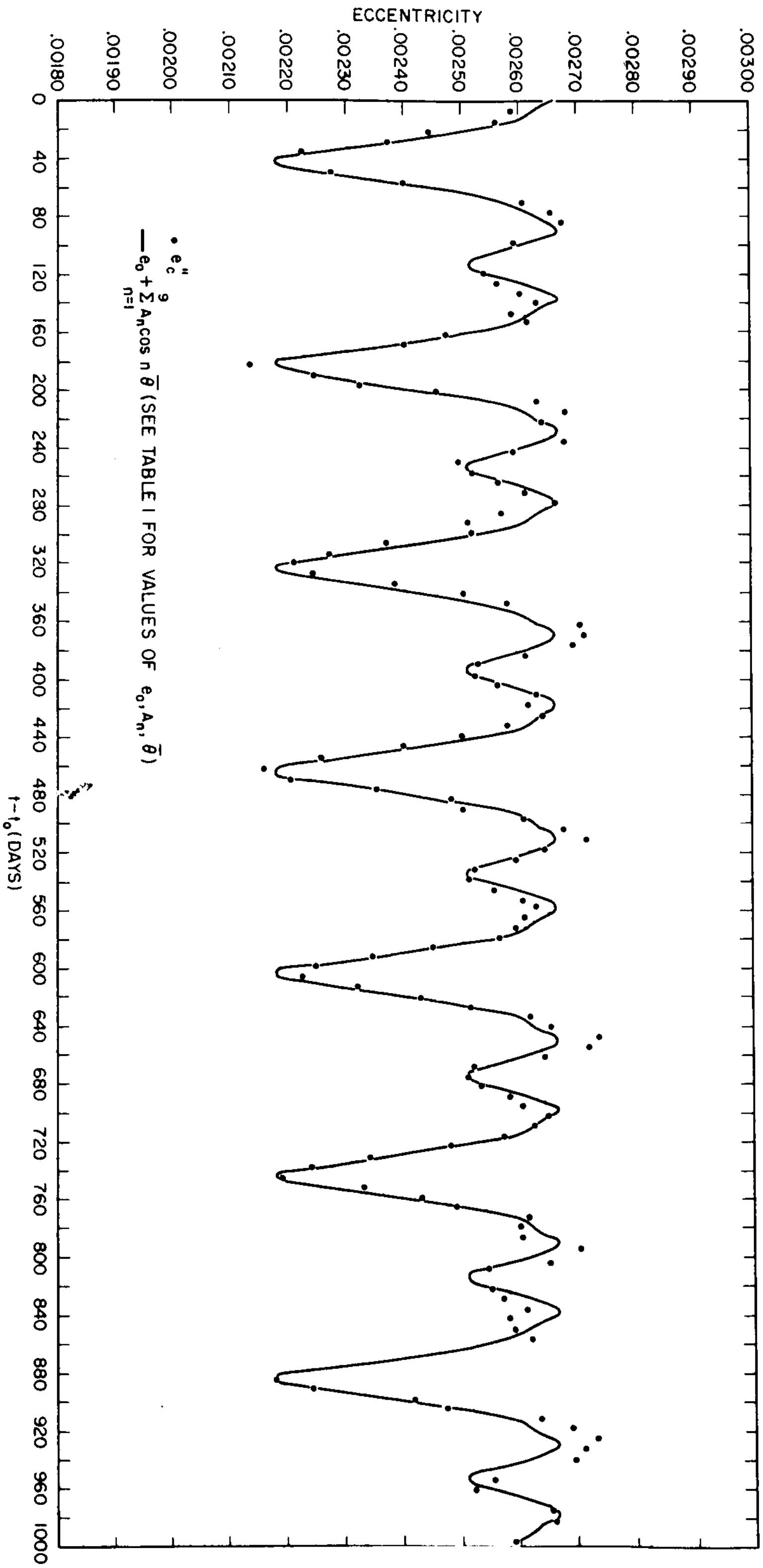

Figure 1—Eccentricity of Alouette 1

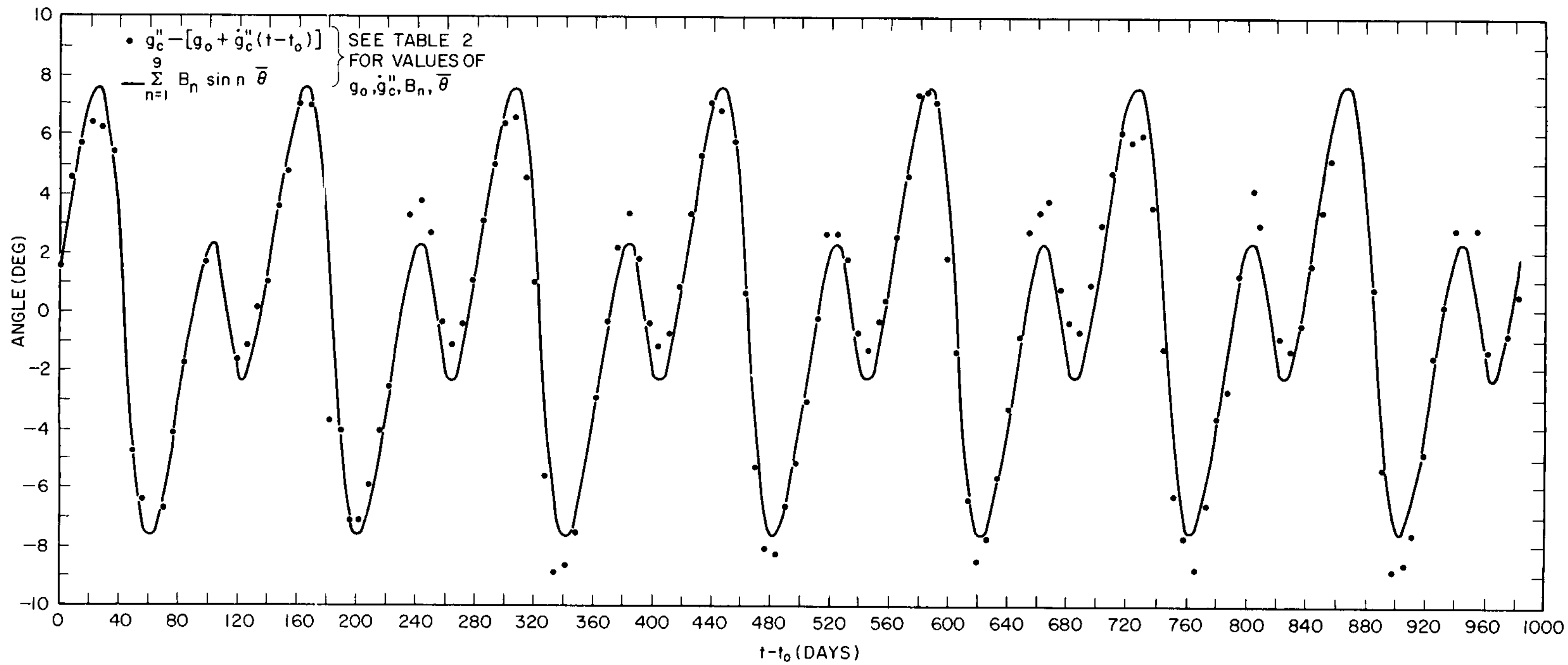

Figure 2–Argument of Perigee of Alouette 1

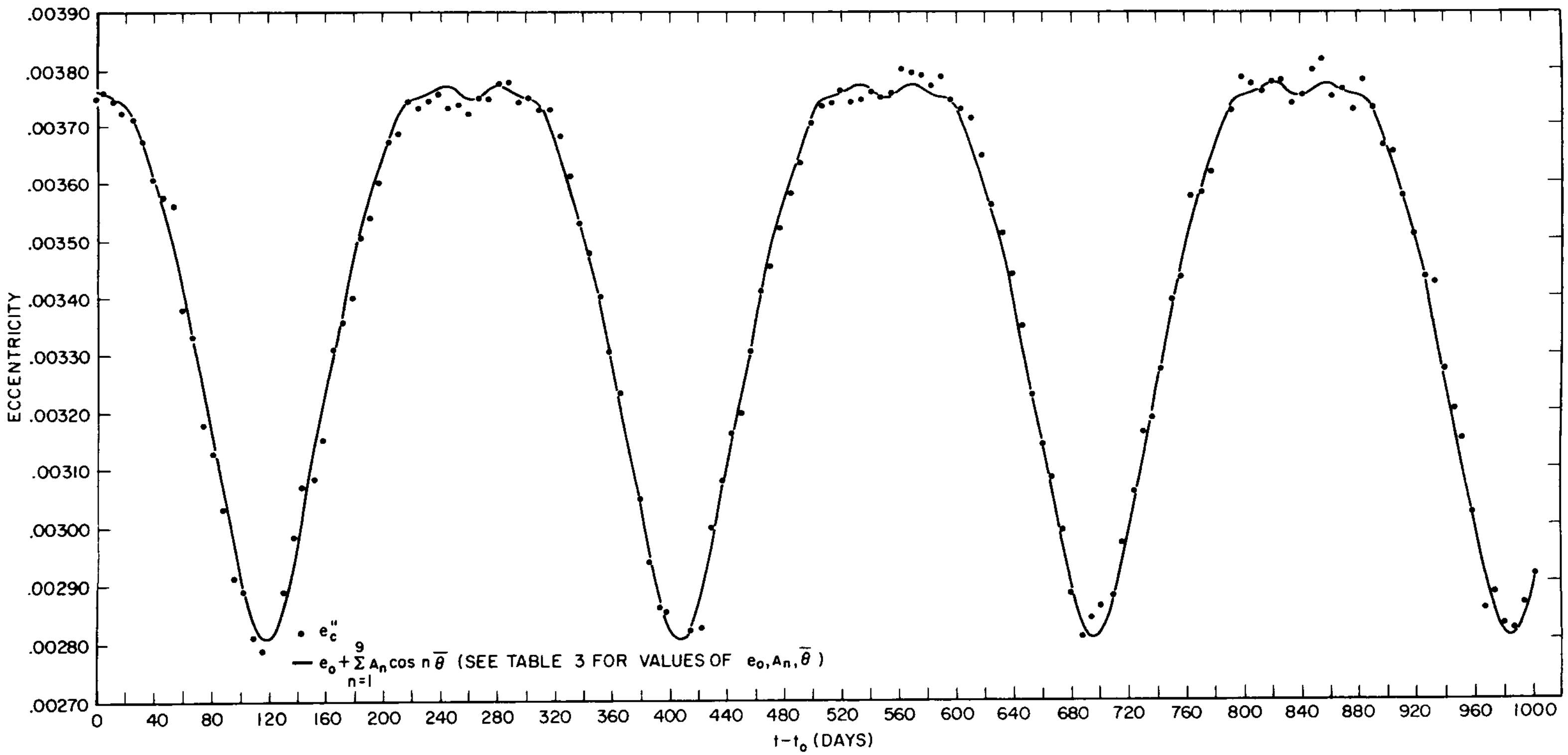

Figure 3– Eccentricity of Tiros 8

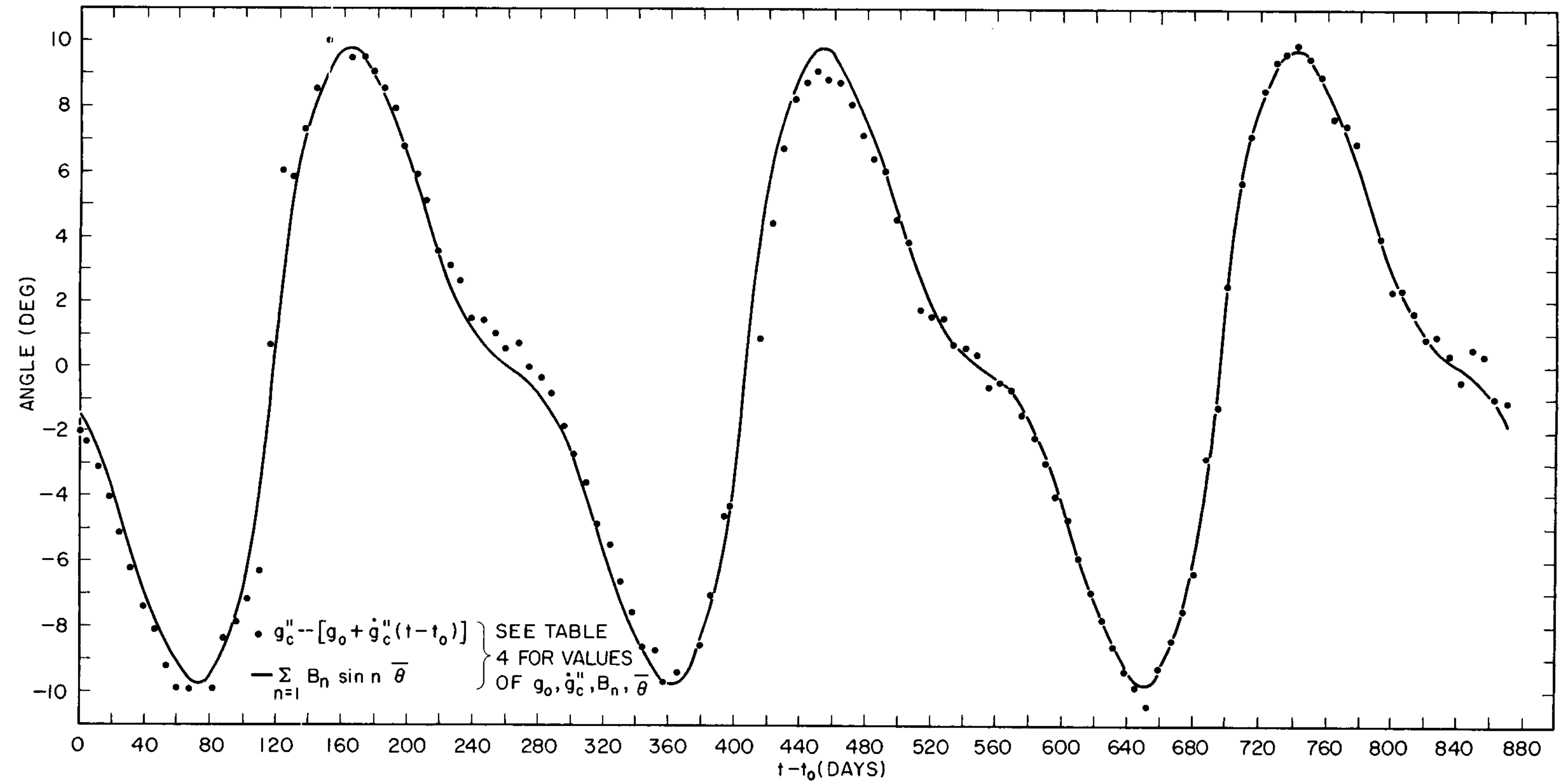

Figure 4–Argument of Perigee of Tiros 8